



Oxford Cambridge and RSA

# AS Level Further Mathematics A

Unit Y535/01 Additional Pure Mathematics

**Tuesday 22 May 2018 – Afternoon**

**Time allowed: 1 hour 15 minutes**



**You must have:**

- Printed Answer Booklet
- Formulae AS Level Further Mathematics A

**You may use:**

- a scientific or graphical calculator

## INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

## INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

1 The points  $A$ ,  $B$  and  $C$  have position vectors  $6\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ ,  $13\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$  and  $16\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$  respectively.

(i) Using the vector product, calculate the area of triangle  $ABC$ . [5]

(ii) Hence find, in simplest surd form, the perpendicular distance from  $C$  to the line through  $A$  and  $B$ . [3]

2 The surface with equation  $z = 6x^3 + \frac{1}{9}y^2 + x^2y$  has two stationary points.

(i) Verify that one of these stationary points is at the origin. [4]

(ii) Find the coordinates of the second stationary point. [5]

3 Given that  $n$  is a positive integer, show that the numbers  $(4n + 1)$  and  $(6n + 1)$  are co-prime. [3]

4 The group  $G$  consists of a set of six matrices under matrix multiplication. Two of the elements of  $G$  are

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix}.$$

(i) Determine each of the following:

- $\mathbf{A}^2$
- $\mathbf{B}^2$  [2]

(ii) Determine all the elements of  $G$ . [4]

(iii) State the order of each non-identity element of  $G$ . [3]

(iv) State, with justification, whether  $G$  is

- abelian
- cyclic. [2]

5 For integers  $a$  and  $b$ , with  $a \geq 0$  and  $0 \leq b \leq 99$ , the numbers  $M$  and  $N$  are such that

$$M = 100a + b \text{ and } N = a - 9b.$$

(i) By considering the number  $M + 2N$ , show that  $17 \mid M$  if and only if  $17 \mid N$ . [4]

(ii) Demonstrate step-by-step how an algorithm based on the result of part (i) can be used to show that 2058376813901 is a multiple of 17. [4]

6 The Fibonacci sequence  $\{F_n\}$  is defined by  $F_0 = 0$ ,  $F_1 = 1$  and  $F_n = F_{n-1} + F_{n-2}$  for all  $n \geq 2$ .

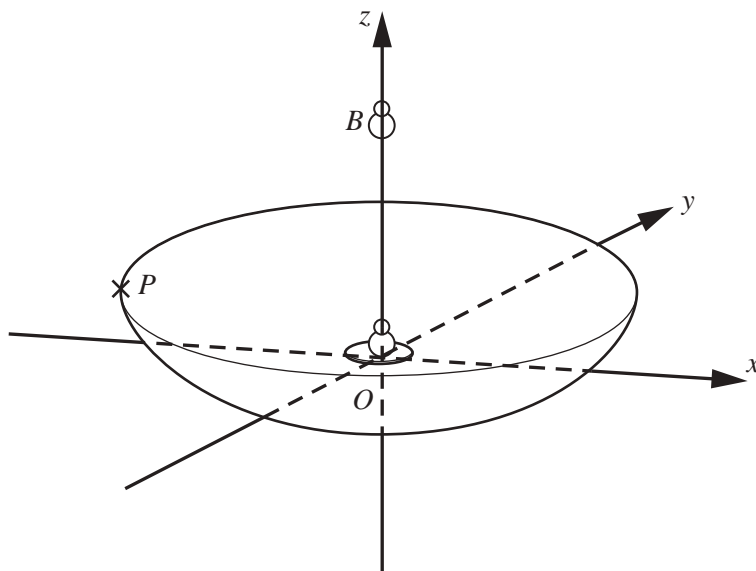
(i) Show that  $F_{n+5} = 5F_{n+1} + 3F_n$  [3]

(ii) Prove that  $F_n$  is a multiple of 5 when  $n$  is a multiple of 5. [5]

- 7 The ‘parabolic’ TV satellite dish in the diagram can be modelled by the surface generated by the rotation of part of a parabola around a vertical  $z$ -axis. The model is represented by part of the surface with equation  $z = f(x, y)$  and  $O$  is on the surface.

The point  $P$  is on the rim of the dish and directly above the  $x$ -axis.

The object,  $B$ , modelled as a point on the  $z$ -axis is the receiving box which collects the TV signals reflected by the dish.



- (i) The horizontal plane  $\Pi_1$ , containing the point  $P$ , intersects the surface of the model in a contour of the surface.
- (a) Sketch this contour in the Printed Answer Booklet. [1]
- (b) State a suitable equation for this contour. [1]
- (ii) A second plane,  $\Pi_2$ , containing both  $P$  and the  $z$ -axis, intersects the surface of the model in a section of the surface.
- (a) Sketch this section in the Printed Answer Booklet. [1]
- (b) State a suitable equation for this section. [1]
- (iii) A proposed equation for the surface is  $z = ax^2 + by^2$ . What can you say about the constants  $a$  and  $b$  within this equation? Justify your answers. [3]
- (iv) The real TV satellite dish has the following measurements (in metres): the height of  $P$  above  $O$  is 0.065 and the perimeter of the rim is 2.652. Using this information, calculate correct to three decimal places the values of
- $a$  and  $b$ ,
  - any other constants stated within the answers to parts (i)(b) and (ii)(b). [4]
- (v) Incoming satellite signals arrive at the dish in linear ‘beams’ travelling parallel to the  $z$ -axis. They are then ‘bounced’ off the dish to the receiving box at  $B$ .
- On the diagram for part (ii)(a) in the Printed Answer Booklet draw some of these beams and mark  $B$ .
  - If the values of  $a$  and  $b$  were changed, what would happen? [2]

**END OF QUESTION PAPER**

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