OCR
Oxford Cambridge and RSA

AS Level Further Mathematics A

Unit Y535/01 Additional Pure Mathematics

Tuesday 22 May 2018 – Afternoon

Time allowed: 1 hour 15 minutes

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You must have:
- Printed Answer Booklet
- Formulae AS Level Further Mathematics A

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

1 The points A, B and C have position vectors $6\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$, $13\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$ and $16\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$ respectively.

(i) Using the vector product, calculate the area of triangle ABC. **[5]**

(ii) Hence find, in simplest surd form, the perpendicular distance from C to the line through A and B. **[3]**

2 The surface with equation $z = 6x^3 + \frac{1}{9}y^2 + x^2y$ has two stationary points.

(i) Verify that one of these stationary points is at the origin. **[4]**

(ii) Find the coordinates of the second stationary point. **[5]**

3 Given that n is a positive integer, show that the numbers $(4n + 1)$ and $(6n + 1)$ are co-prime. **[3]**

4 The group G consists of a set of six matrices under matrix multiplication. Two of the elements of G are

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix}.$$

(i) Determine each of the following:

- $\mathbf{A}^2$
- $\mathbf{B}^2$

[2]

(ii) Determine all the elements of G. **[4]**

(iii) State the order of each non-identity element of G. **[3]**

(iv) State, with justification, whether G is

- abelian
- cyclic.

[2]

5 For integers a and b, with $a \geqslant 0$ and $0 \leqslant b \leqslant 99$, the numbers M and N are such that

$$M = 100a + b \quad \text{and} \quad N = a - 9b.$$

(i) By considering the number $M + 2N$, show that $17 \mid M$ if and only if $17 \mid N$. **[4]**

(ii) Demonstrate step-by-step how an algorithm based on the result of part **(i)** can be used to show that $2\,058\,376\,813\,901$ is a multiple of 17. **[4]**

6 The Fibonacci sequence $\{F_n\}$ is defined by $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for all $n \geqslant 2$.

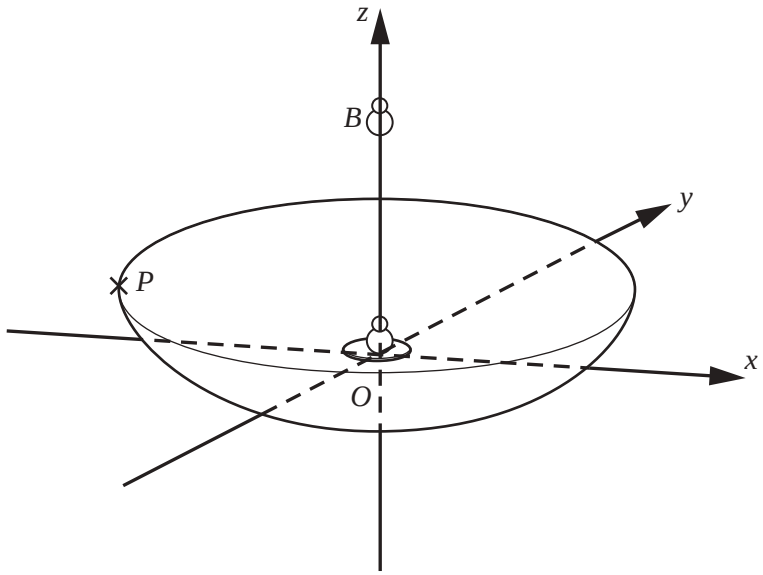
(i) Show that $F_{n+5} = 5F_{n+1} + 3F_n$ **[3]**

(ii) Prove that F_n is a multiple of 5 when n is a multiple of 5. **[5]**

7 The 'parabolic' TV satellite dish in the diagram can be modelled by the surface generated by the rotation of part of a parabola around a vertical z-axis. The model is represented by part of the surface with equation $z = f(x, y)$ and O is on the surface.

The point P is on the rim of the dish and directly above the x-axis.

The object, B, modelled as a point on the z-axis is the receiving box which collects the TV signals reflected by the dish.



(i) The horizontal plane Π_1, containing the point P, intersects the surface of the model in a contour of the surface.

(a) Sketch this contour in the Printed Answer Booklet. **[1]**

(b) State a suitable equation for this contour. **[1]**

(ii) A second plane, Π_2, containing both P and the z-axis, intersects the surface of the model in a section of the surface.

(a) Sketch this section in the Printed Answer Booklet. **[1]**

(b) State a suitable equation for this section. **[1]**

(iii) A proposed equation for the surface is $z = ax^2 + by^2$. What can you say about the constants a and b within this equation? Justify your answers. **[3]**

(iv) The real TV satellite dish has the following measurements (in metres): the height of P above O is 0.065 and the perimeter of the rim is 2.652. Using this information, calculate correct to three decimal places the values of

- a and b,
- any other constants stated within the answers to parts **(i)(b)** and **(ii)(b)**. **[4]**

(v) Incoming satellite signals arrive at the dish in linear "beams" travelling parallel to the z-axis. They are then 'bounced' off the dish to the receiving box at B.

- On the diagram for part **(ii)(a)** in the Printed Answer Booklet draw some of these beams and mark B.
- If the values of a and b were changed, what would happen? **[2]**

END OF QUESTION PAPER